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UDK 517.981.1+519.216.22

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SURFACE MEASURES AND THE STOKES FORMULA IN LOCALLY CONVEX SPACES

(specialty - 01.01.01 - mathematical analysis)

ABSTRACT
of the Dissertation submitted for the academic degree
of Candidate of Physical and Mathematical Sciences[1]

Moscow, 2005

[1]The academic degree of Candidate of Sciences corresponds to the Ph.D. degree

This work has been performed at the department of Theory of Functions and Functional Analysis of the Faculty of Mechanics and Mathematics at Lomonosov Moscow State University.

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The dissertation talk will take place on May 20th, 2005, at 16:15 at the session of the Dissertation Committee D.501.001.85 at Lomonosov Moscow State University at the address: GSP–2, Leninskije gory, MSU, Faculty of Mechanics and Mathematics, auditorium 16–24, 119992, Moscow

The Dissertation is available at the library of the Faculty of Mechanics and Mathematics of MSU (main building, 14 floor).

The abstract is sent out on April 20th, 2005

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[2]The academic degree of Doctor of Sciences corresponds to the Habilitation

Description of the thesis

Relevance of the topic

The Ph.D. thesis belongs to infinite dimensional analysis. Two classes of surface measure on locally convex spaces are considered. The first class consists of surface measures on (infinite dimensional) manifolds of locally convex spaces possessing a finite dimension. Here it is assumed that the surface measures are generated by smooth measures on the surrounding spaces. The second class consists of surface measures on (infinite dimensional) manifolds possessing an infinite codimension. Here as a surrounding space we consider the space of continuous factions defined on a square and taking values in a Euclidian space. It is assumed that a measure generated by a so called Brownian sheet is given in the surrounding space. As a submanifold we consider the totality of continuous functions defined on the same square and taking values in a compact Riemannian manifold of this Euclidian space. We also prove an analog of Chernoff's theorem for evolution families of operators.

The investigation of the surface measures of the first class is one of the traditional directions of infinite dimensional analysis. It closely related to investigation of finite dimensional differential operators and the general problem of desintegrating of measures. Studying of these surface measures has started in works of A. V. Skorokhod [3] and A. V. Uglanov [4] about 30 years ago within the theory of smooth measures on infinite dimensional spaces created in works of S. V. Fomin, O. G Smolyanov and their students. The theory of surface measures is essentially used in the Malliavin calculus [5] [6]. Nowadays this area of infinite dimensional analysis can be considered as classical.

Nevertheless, the technique developed for investigation of surface measures on submanifolds of a finite codimension turned out not to be sufficient for investigation of surface measures on submanifolds possessing simultaneously an infinite dimension and an infinite codimension. The appearing difficulties were overcome in the series of works of O. G. Smolyanov, H. v. Weizsäcker and

[3] Skorokhod, A. V., Integration in Hilbert space, Moscow, 1995

[4] Uglanov, A. V. Surface intergrals in the Banach space. Mat.Sb. [Math. USSR-Sb], 110 (1979), No 2, c.189–217.

[5] Malliavin P., Hypoellipticity in infinite dimensions, Diffusion processes and related problems in analysis., Vol.1 (Evanston, IL, 1989), 1990, pp. 17-31.

[6] H. Airault, P. Malliavin, Integration on loop groups. II. Heat equation for the Wiener measure, J. Funct. Anal. **104** (1992), No 1, 71-109

their co-authors [7] [8] [9] [10]. In these works a technique of construction of surface measures on submanifolds of the vector space of functions of one real variable, taking values in $\mathbb{R}^n$, was developed. It was assumed that the manifolds consist of functions taking values in a Rimannian submanifold of $\mathbb{R}^n$. The obtained results are related to studying evolutional differential equations on manifolds. The next natural step is generalizing this technique to the case of a vector space consisting of functions of several real variables and its submanifold. [11] (see also [12] [13] [14] [15] [16]). Manifolds of this kind appear in quantum field theory and M-theory. Thus, the topic of the Ph.D. thesis appears to be quite actual.

Scientific novelty

All the results of the Ph.D. thesis are new. The main results (1–4) are as follows:

1. We describe a method of approximation of the Uglanov surface measures by "volume" measures on properly defined surface layer subsets, and prove the surface layer theorem. The surfaces under consideration are submanifolds of codimension 1 in a locally convex space.

2. We develop a calculus of differential forms of a finite codegree on a locally convex space, and for surfaces of codimension 1 in a locally convex space we prove the Stokes formula.

3. We obtain an analog of Chernoff's theorem for evolution families of operators.

[7] Smolyanov O. G., Surface measures on loop groups, Doklady mathematics, 1995

[8] Smolyanov O.G., Weizsäcker H.v., Wittich O., Brownian motion on a manifold as a limit of stepwise conditioned standard Brownian motions, Canadian Mathematical Society, Conference Proceedings, Vol. 29, 2000, pp. 589-602

[9] Smolyanov, O.G.; von Weizsäcker, H.; Wittich, O.; Sidorova, N.A. Surface measures on trajectories in Riemannian manifolds generated by diffusions. (Russian, English) Dokl. Math. 63, No.2, 203-207 (2001); translation from Dokl. Akad. Nauk, Ross. Akad. Nauk 377, No.4, 441-446 (2001).

[10] Sidorova, Nadezda A.; Smolyanov, Oleg G.; von Weizsäcker, Heinrich; Wittich, Olaf The surface limit of Brownian motion in tubular neighborhoods of an embedded Riemannian manifold. (English) J. Funct. Anal. 206, No.2, 391-413 (2004)

[11] Smolyanov O.G., Weizsäcker H.v., Wittich O., Construction of diffusions on the totality of mappings of an interval into a compact Riemannian manifold, Doclady Mathematics v.402, N 6, pp.1-5, 2005

[12] Driver B.K., Integration by parts and quasi-invariance for heat kernel measures on loop groups, J. Funct. Anal. 149 (2) (1997) 470-547.

[13] Driver B.K., Srimurthy V., Absolute continuity of heat kernel measure with respect to pinned Wiener measure on loop groups, The Annals of Probability, 2000.

[14] Malliavin P., Hypoellipticity in infinite dimensions, Diffusion processes and related problems in analysis., Vol.1 (Evanston, IL, 1989), 1990, pp. 17-31.

[15] H. Airault, P. Malliavin, Integration on loop groups. II. Heat equation for the Wiener measure, J. Funct. Anal. **104** (1992), No 1, 71-109

[16] Srimurthy, Vikram K., On the equivalence of measures on loop space, Probab. Theory Relat. Fields 118, No.4, 522-546 (2000)

4. We describe a method of construction of a Brownian sheet with values in a compact Riemannian manifold embedded into a finite dimensional Euclidian space. This result essentially generalizes an analogous result of P. Malliavin for Lie groups.

Investigation methods

We use methods of infinite dimensional and stochastic analysis, and a series of special constructions.

Theoretical and practical value

The thesis has a theoretical perspective. Its results can be used for problems arising in stochastic analysis on manifolds, in particular for investigation of stochastic fields with values in a compact Riemannian manifold.

Approbation of the Ph.D. thesis

The results of the Ph.D. thesis were presented at the seminar "Infinite dimensional analysis and mathematical physics" headed by Professor O. G. Smolyanov and Professor E. T. Shavgulidze at the Faculty of Mechanics and Mathematics of MSU, at the seminar of academiian V. S. Vladimirov and corresponding member of the Russian Academy of Sciences I. V. Volovich at the department of mathematical physics of Steklov mathematical institute of the Russian Academy of Sciences, and at the XXV conference "Lomonosov readings" at MSU (2003).

Publications

The main contents of the thesis has been published in 3 papers of the author. The author does not have papers in coauthorship.

Organization and amount of work

The thesis consists of introduction and 3 main chapters divided into paragraphs. Number of pages is 101. The list of references contains 40 items.

Brief contents of the thesis

Introduction

In the introduction we formulate the main results of the Ph.D. thesis and give a review of the literature on the topic of the thesis.

Chapter 1

In this chapter we consider surface measures on submanifolds of a finite codimension and prove the surface layer theorems. Everywhere in this chapter E is a locally convex space, H its Hilbert subspace, H is dense in E and the identical mapping of H into E is continuous.

Definition 1. *Let $D_F \subset E$ be an open set and $F : D_F \to \mathbb{R}$ be a continuous function having a derivative $F' : D_F \to H' \cong H$ along the subspace H. Let $S \stackrel{\text{def}}{=} \{x \in D_F : F(x) = 0\}$, and $F'(x) \neq 0$ for all $x \in S$. The vector $n^S = \frac{F'(x)}{\|F'(x)\|}$ (the linear continuous on H functional $F'(x)$ we identify with an element of H) we call the normal vector to S at the point x.*

Let the set S be such as in Definition 1, and $B \subset S$ be a subset. For all $\varepsilon > 0$ we define a set

$$B^\varepsilon \stackrel{\text{def}}{=} \{x + tn^S(x) \ : \ x \in B \ , \ t \in (-\varepsilon, \varepsilon)\} \ .$$

Definition 2. *We call the set G a surface, if the following assumptions are fulfilled (1-3):*
Assumption 1. *The set G is a graph of a continuous bounded function f, given on an open subset $\mathcal{U}$ of a closed space T_b of codimension 1 of the space E, and taking values on the line $R_b = \{tb : t \in \mathbb{R}\}$ where $b \in H$ is a unit vector orthogonal in H to the subspace $H_b \stackrel{\text{def}}{=} T_b \bigcap H$.*

Remark 1. We denote by P_b the projector operator on the subspace T_b along the vector b.

Assumption 2. *The function f has a continuous first derivative $f' : \mathcal{U} \to H_b' \cong H_b$, a continuous second derivative, $f'' : \mathcal{U} \to \mathcal{L}(H_b, H_b') \cong \mathcal{L}(H_b, H_b)$ and a third derivative $f''' : \mathcal{U} \to \mathcal{L}(H_b, \mathcal{L}(H_b, H_b)) \cong \mathcal{L}(H_b, \mathcal{L}(H_b, H_b))$ along the subspace H; and there exists a constant $K_f > 0$ such that*

$$\|f'(x)\| \leqslant K_f \ , \quad \|f''(x)\|_{\mathcal{L}(H_b,H_b)} \leqslant K_f \ , \quad \|f'''(x)\|_{\mathcal{L}(H_b,\mathcal{L}(H_b,H_b))} \leqslant K_f \ , \quad x \in \mathcal{U} \ ,$$

where $\|\cdot\|_{\mathcal{L}(H_b,H_b)}$ and $\|\cdot\|_{\mathcal{L}(H_b,\mathcal{L}(H_b,H_b))}$ are the operator norms in the corresponding spaces.
Assumption 3. *There exists $\varepsilon_b > 0$ such that the mapping g_f can be extended to one-to-one mapping*

$$\varphi \ : \mathcal{U} \times (-\varepsilon_b, \varepsilon_b) \to E \ , \quad (x,t) \mapsto g_f(x) + tn^G(x),$$

and the inverse mapping $\psi = \varphi^{-1}$ defined on G^{ε_b}, is continuous.

A measure on a topological space X is understood to be a σ-additive function defined on a Borelean σ-algebra $\mathfrak{B}_X$ of Borelean subsets of the space X.

The differentiability of measures means τ_s-differentiability [17].

Let ν be a non-negative Radon measure, defined on E, two times differentiable along the space H; $\nu_{_G}$ be a surface measure on G defined in [18].

According to [17], on $\mathfrak{B}_G$ the function $\nu_{_G}$ is given as follows:

$$\nu_{_G}(Q) = \int\limits_Q \frac{\nu^d(dx)}{(n^G(P_b x), b))}$$

where ν^d is a measure on G given by $\nu^d(Q) = d_b\nu(\bigcup\limits_{s\leqslant 0}\{Q+sb\}),\ Q \in \mathfrak{B}_G$. In this case $\nu_{_G}$ is a (non-negative) measure on G [17].

Let $B \subset G$ be a subset. By Assumption 3, for every $\varepsilon \leqslant \varepsilon_b$, the set B^ε is open if the set B is open, and the set B^ε is Borelean if the set B is Borelean.

Let $\nu_{_T}$ be the projection of ν onto T_b along R_b. We say that a Borelean subset $B \subset G$ possesses the property $(*)$ if $\nu_{_T}(\partial P_b B) = 0$.

THEOREM 1. *Let $B \subset G$ be an open subset which is contained in G together with its closure, and possesses the property $(*)$. Then,*

$$\nu_{_G}(B) = \lim_{\varepsilon\to 0} \frac{\nu(B^\varepsilon)}{2\varepsilon}\ . \tag{1}$$

Outline of proof. On $\mathcal{U} \times \mathbb{R}$ we define the function

$$f(x,\varepsilon) = f(x) + \frac{\varepsilon}{(n^G(x), b)}\ .$$

Let $B \subset G$ be an open set which possesses the property $(*)$. Define $f_\varepsilon(x) = f(x,\varepsilon)$, $B_{f,f_\varepsilon} \overset{\text{def}}{=} \{x+tb\ ,\ x \in P_b B\ ,\ f(x) \leqslant t < f_\varepsilon(x)\}$. By the results of the paper of Uglanov [19]

$$\lim_{\varepsilon\to 0} \varepsilon^{-1}\nu(B_{f,f_\varepsilon}) = \int\limits_B (n^G(x), b)\frac{\partial f_\varepsilon}{\partial\varepsilon}(x,0)\nu_{_G}(dx) = \nu_{_G}(B)\ .$$

We prove that $\nu(B_\varepsilon \triangle B_{f,f_\varepsilon}) = o(\varepsilon)$ which will imply the formula (1). □

[17] *Daletskij, Yu.L.; Fomin, S.V.*, Measures and differential equations in infinite-dimensional space. (English) Mathematics and Its Applications. Soviet Series. 76. Dordrecht etc.: Kluwer Academic Publishers. xv, 337 p. (1991).

[18] *Uglanov A. V.*, Surface integrals in linear topological spaces. (Russian, English) Dokl. Math. 52, No.2, 227-230 (1995); translation from Dokl. Akad. Nauk, Ross. Akad. Nauk 344, No.4, 450-453 (1995).

[19] *Uglanov A. V.*, Surface integrals in a Banach space. (English) Math. USSR, Sb. 38, 175-199 (1981).

THEOREM 2. *Let the family of functions* $f_B(\varepsilon) = \dfrac{\nu^\varepsilon(B)}{\nu_G(B)}$, *where* $B \in \mathfrak{B}(G)$, *be uniformly bounded on the interval* $(0, \varepsilon_b]$. *Then, the formula* (1) *holds for every Borelean subset of the surface* G.

Outline of proof. The proof of the theorem is obtained using the fact that the measures ν_G and ν possess the Radon property. □

Chapter 2

In this chapter, we prove the Stokes formula for differential forms of a finite codegree and for submanifolds of a finite codimension in a locally convex space.

Let Ξ_n $(n \in N)$ denote a vector space of all differentiable along the subspace H differential forms of degree n such that their differentials are continuous, and let, in addition to this, the ranges of values of differential forms from Ξ_n and their differentials be bounded. By S_n, we denote the space of all differential forms of codegree n differentiable along the subspace H which are Radon measures and assume that the differential forms from S_n and their differentials are measures of a bounded variation. We denote by $\bar{S}_n$ and $\bar{\Xi}_n$ $(n \in \mathbb{N})$ pseudo-topological vector spaces of all linear continuous functionals on Ξ_n and S_n, respectively. We assume that $\bar{S}_n$ and $\bar{\Xi}_n$ contain S_n and Ξ_n as dense subsets. Let V be a domain in the space E such that its boundary ∂V can be covered by a union of a finite number of surfaces (by a surface, we understand the object defined in the previous paragraph) $\mathcal{U}_i$ of codimension 1. Assume that the indicator $\mathbb{I}_V$ of the set V is an element of the space $\bar{\Xi}_0$.

THEOREM 3. *Let* $\nu \in S_0$. *Then* $d\,\mathbb{I}_V \cdot \nu$ *is an* H*-valued Radon measure on* X *concentrated on* ∂V. *Moreover, if* $\nu^{\partial V}$ *is a surface measure* ∂V *(in terms of the definition of Uglanov's paper*[18]*) generated by the measure* ν, *then* $n^{\partial V} \cdot \nu^{\partial V} \in \bar{S}_1$, *and the measures* ν *and* $\nu^{\partial V}$ *are related through the identity:*

$$d\,\mathbb{I}_V \cdot \nu = -n^{\partial V} \cdot \nu^{\partial V}.$$

DEFINITION 3. *Let* $\omega \in S_1$ *and* $\omega^{\partial V} \in \bar{S}_1$. *We define the integral of* ω *over the surface* ∂V *by the identity:*

$$\int_{\partial V} \omega = \int_{\partial V} (n^{\partial V}, \omega^{\partial V}(dx)).$$

THEOREM 4 (The Stokes formula). *Let* $\omega \in S_1$ *and* $\omega^{\partial V} \in \bar{S}_1$, *then*

$$\int_{\partial V} \omega = \int_V d\omega.$$

Outline of proof. The Stokes formula can be proved applying Theorem 3. □

Chapter 3

In this chapter we investigate surface measures on manifolds of an infinite codimension. Here the manifolds are totalities of continuous functions defined on a square and taking values in a compact Riemannian submanifold of a Euclidian space; the surrounding spaces consist of continuous functions defined on the same square and taking values in the same Euclidian space. It is assumed that the measure on the surrounding Euclidian space is generated by a so called Brownian sheet. One can consider the obtained result as a construction of a Brownian sheet with values in a compact Riemannian manifold. This result essentially generalizes an analogous result of P. Malliavin for Lie groups [20]. In this chapter we also prove an analog of Chernoff's theorem for evolution families of operators, and with the help of this result we construct a non-homogeneous process on a compact Riemannian manifold.

Let A_t be generators of strongly continuous semigroups on a Banach space E such that the space $F = \cap_t D(A_t)$ is dense in E. Assume that for all $x \in F$ $\sup_{S \leqslant t \leqslant T} \|A_t x\|_E < \infty$. Introduce the norm in the space F: $\|x\|_F = \|x\|_E + \sup_t \|A_t x\|_E$. F is a Banach space relative to this norm and all the operators $A_t : F \to E$ are continuous.

Consider nonautonomous Cauchy problem

$$\begin{cases} \dot{u}(t) = A_t u(t) \\ u(s) = x \ , \end{cases} \tag{2}$$

where $t, s \in [S, T]$, $s \leqslant t$. Further we will assume that the Cauchy problem (2) is well-posed [21] (There exist several sufficient conditions of well-posedness of the Cauchy problem (2). References to the corresponding literature one can find for example in [20]). According to [20] in this case for the Cauchy problem (2) there exist a strongly continuous evolution family $U(t, s)$, $s, t \in [S, T], s \leqslant t$ solving the Cauchy problem.

Consider another nonautonomous Cauchy problem

$$\begin{cases} \dot{u}(t) = -\bar{A}_t u(t) \\ u(r) = x \end{cases} \tag{3}$$

for $t \leqslant r$ and $\bar{A}_t = A_{S+T-t}$.

LEMMA 1. *The Cauchy problem (3) is well-posed if and only if the Cauchy problem (2) is well-posed. In addition to this, the evolution family $U(t, r)$ $t \leqslant r$ solving the Cauchy problem (3) satisfies the following identity $U(t_1, t_2)U(t_2, t_3) = U(t_1, t_3)$ which holds for all $t_1 \leqslant t_2 \leqslant t_3$.*

[20] Malliavin P., Hypoellipticity in infinite dimensions, Diffusion processes and related problems in analysis., Vol.1 (Evanston, IL, 1989), 1990, pp. 17-31.

[21] *Engel, Klaus-Jochen; Nagel, Rainer*, One-parameter semigroups for linear evolution equations. (English) Graduate Texts in Mathematics. 194. Berlin: Springer. xxi, 586 p., 2000.

THEOREM 5. *Let A_t be generators of strongly continuous semigroups, B be another generator of a strongly continuous semigroup. Assume that the following assumptions are fulfilled*

1. *the space $G = F \cap D(B)$ is dense in E;*
2. *for every $x \in F$ the functions $[S,T] \to E$, $t \mapsto A_t x$ are continuous;*
3. *the operator B commutes with every operator A_t;*
4. *$\{A_t\}$ is a stable system of generators* [22] *;*
5. *the Cauchy problem (3) is well-posed on F; let $U(t,r)$ be the evolution family solving the Cauchy problem (3);*
6. *there exists a dense in E subset $D \subset G$ such that for all t and r holds: $U(t,r)D \subset D$, $T_r(t)F \subset D$;*
7. *for every fixed r and $x \in D$ the family of functions $s \mapsto A_t U(s,r)\,x$, $s \mapsto BU(s,r)\,x$ are uniformly continuous;*
8. *for every $y \in F$ the family of functions $s \mapsto A_{t+s}T_t(s)\,y$ is uniformly continuous at the point $s = 0$.*

Let Q_{t_1,t_2}, $t_1, t_2 > 0$ be a two-parameter family of linear contractions on E, such that $\frac{Q_{\tau,\tau+\Delta\tau}-I}{\Delta\tau}\, e^{aB}x \to A_\tau\, e^{aB}x$ as $\Delta\tau \to 0$ for all $x \in D$, $a > 0$, and uniformly in τ. Let $S \leqslant s < t \leqslant T$ and $\{s = t_0, t_1, \ldots, t_n = t\}$ be a partition of the interval $[s,t]$ such that $\max \Delta t_j \to 0$ as $n \to \infty$ where $\Delta t_j = t_{j+1} - t_j$. Then for all $x \in E$

$$Q_{t_0,t_1} \ldots Q_{t_{n-1},t_n} x \to U(s,t)\,x \ ,$$

as $n \to \infty$.

Let M be a compact Riemannian manifold of dimension d without boundary, isometrically imbedded into $\mathbb{R}^m$. Let $\mathcal{P}_1 = \{t_0 = 0 \leqslant t_1 \leqslant \ldots \leqslant t_n = 1\}$ be a partition of the interval $[0,1]$, $\varphi : [0,1] \to M - x$ be a differentiable function such that $\varphi(0) = 0$. If E is a locally convex space, then E^t denotes $C([0,t],E)$ - the space of all continuous functions on $[0,t]$ with values in E. Every $\omega \in E^1$ can be identified with a sequence of n elements $(\omega_1, \omega_2, \ldots, \omega_n) \in E^{t_1} \times E^{t_2-t_1} \times \ldots \times E^{t_n-t_{n-1}}$ where ω_j is defined on the interval $[0, t_j - t_{j-1}]$ by the formula $\omega_j(t) = \omega(t_{j-1} + t)$. Define the function $\varphi_{t_{i-1}t_i}$ on the interval $[0, t_i - t_{i-1}]$ by the formula

$$\varphi_{t_{i-1}t_i}(t) = \varphi(t_{i-1} + t) - \varphi(t_{i-1}) \ .$$

[22] *Tanabe, Hiroki*, Equations of evolution. Translated from Japanese by N. Mugibayashi and H. Haneda. (English) Monographs and Studies in Mathematics. 6. London - San Francisco - Melbourne: Pitman. XII, 1979, 260 p.

Further, consider a process $(\mathbf{W}^z_{\psi,s})_t = \psi(t) + B^s_t$ where $\psi : [0,1] \to \mathbb{R}^m$ is a continuous function satisfying the condition $\psi(0) = 0$, and B^s_t is a multiple Brownian motion with the parameter s, starting at the point z.

Let $\mathbb{W}^z_{\psi,s}$ denote the distribution of the process $(\mathbf{W}^z_{\psi,s})_t$, $\mathbb{E}_{z,\psi,s}$ denote the expectation relative this distribution.

In the following construction of a Brownian sheet on the manifold we will prove the existence of the limit

$$\int_{C([0,t],\mathbb{R}^m)} f(\omega)\mathbb{W}^z_{M,\psi,s,t}(d\omega) = \lim_{\varepsilon\to 0} \frac{\mathbb{E}_{z,\psi,s}\{f(\omega)\,\mathbb{I}_{\{(\mathbf{W}^z_{\psi,s})_t\in U_\varepsilon(M)\}}\}}{\mathbb{W}^z_{\psi,s}\{(\mathbf{W}^z_{\psi,s})_t \in U_\varepsilon(M)\}}$$

relative to the family of bounded continuous cylinder functions. This limit defines a measure $\mathbb{W}^z_{M,\psi,s,t}$. Using this measure we define the measure $\mathbb{W}^x_{M,\varphi,s,\mathcal{P}_1}$ in the following way:

$$\int\limits_{C([0,1],\mathbb{R}^m)} h(\omega)\mathbb{W}^x_{M,\varphi,s,\mathcal{P}_1}(d\omega) = \int\limits_{C([0,t_1],\mathbb{R}^m)} \mathbb{W}^x_{M,\varphi_{0t_1},s,t_1}(d\omega_1) \int\limits_{C([0,t_2-t_1],\mathbb{R}^m)} \mathbb{W}^{\omega_1(t_1)}_{M,\varphi_{t_1t_2},s,t_2-t_1}(d\omega_2) \dots \int\limits_{C([0,t_n-t_{n-1}],\mathbb{R}^m)} \mathbb{W}^{\omega_{n-1}(t_{n-1}-t_{n-2})}_{M,\varphi_{t_{n-1}t_n},s,t_n-t_{n-1}}(d\omega_n)h(\omega_1,\omega_2,\dots,\omega_n). \tag{4}$$

Theorem 6. *Let φ be a differentiable function. Then, as the mesh of $\mathcal{P}_1$ tends to zero, the sequence of measures $\mathbb{W}^x_{M,\varphi,s,\mathcal{P}_1}$ convergence weakly relative to the family of cylindric functions.*

Outline of proof. The Chernoff theorem for evolution families is applied to the family of generators $A_t f = (\varphi'(t), \nabla_M f)_{\mathbb{R}^m} - \frac{s}{2}\Delta_M$. □

Proposition 1. *Let $\imath$ be an isometric imbedding of the manifold M into $\mathbb{R}^m$, $g \in C^2(M)$. Then*

$$\frac{1}{(2\pi t)^{\frac{d}{2}}}\int_M g(z)e^{-\frac{|z-y|^2}{2t}}\lambda_M(dz) = g(y)+\frac{t}{8}\,g(y)\big(c(y)-scal(y)\big)-\frac{t}{2}\Delta_M g(y)+tR(t,y)\ ,$$

where $|R(t,y)| < Kt^{1/2}$, K is a constant which does not depend on y, $scal(y)$ is the scalar curvature at the point y; the function $c(y)$ has the form: $c(y) = \sum_{k,l}\sum_{\alpha}\Big(\frac{\partial^2 i^\alpha}{\partial x^k\partial x^l}\Big)^2(0)$, where x^k are normal coordinates in a neighborhood U_y of the point y provided by a homeomorphism of the neighborhood U_y onto a neighborhood of zero U in $\mathbb{R}^d$; i is the mapping $\imath$ in coordinates x^k. Independently of local coordinates $c(y)$ can be written as follows $c(y) = -\frac{1}{2}\Delta_M\Delta_M\,|y-\cdot|^2\big|_y - \frac{1}{3}\,scal(y)$, and, hence, $c(y)$ depends only on the imbedding $\imath$.

About the proof. A more precise asymptotic than the asymptotic from the paper of Smolyanov, Weizsäcker and Wittich [23]. s obtained. Outline of proof is the same. □

COROLLARY 1. *Let $g \in C^2(M)$. Then the following asymptotic holds:*

$$\int_M g(z) e^{-\frac{|z-y|^2}{2t}} \lambda_M(dz) \Big/ \int_M e^{-\frac{|z-y|^2}{2t}} \lambda_M(dz) = g(y) - \frac{t}{2}\Delta_M g(y) + tR_1(t,y),$$

where $|R_1(t,y)| < K_1 t^{1/2}$, K_1 is a constant which does not depend on y.

PROPOSITION 2. *Let $\imath$ be an isometric imbedding of the manifold M into $\mathbb{R}^m$, $g \in C^2(M)$, $y \in M$, $0 < t < t_1 < 1$, u_1 and u_2 are such that $|u_2 - u_1| < t_1^\alpha$, $\alpha > 0$. Further let Pr_M be mapping of projection onto the manifold M along the subspaces normal to the manifold which are defined in the proper neighborhood of the manifold, $(u_2-u_1)_M^y$ and $(u_2-u_1)_\perp^y$ are such that:*

$$(u_2-u_1)_M^y = \mathrm{Pr}_M(y+u_2-u_1) - y, \quad (u_2-u_1)_\perp^y = y+u_2-u_1-\mathrm{Pr}_M(y+u_2-u_1).$$

Then the following asymptotic holds:

$$\frac{\int_M g(z) e^{-\frac{|z-y-(u_2-u_1)|^2}{2t}} \lambda_M(dz)}{\int_M e^{-\frac{|z-y-(u_2-u_1)|^2}{2t}} \lambda_M(dz)} = g(y+(u_2-u_1)_M^y) - \frac{t}{2}\Delta_M g(y+(u_2-u_1)_M^y)$$
$$+ g(y+(u_2-u_1)_M^y)\mathcal{R}(y,u_2-u_1) + tR_2(t,t_1,y,u_2-u_1); \quad (5)$$

for the rest terms, we have: $|R_2(t,t_1,y,u_2-u_1)| < K_2 t_1^\alpha$ (K_2 is a constant),

$$\mathcal{R}(y,u_2-u_1) = -\frac{1}{4}(\Delta_M \imath(y+(u_2-u_1)_M^y),(u_2-u_1)_\perp^y)^2$$
$$+ \sum_{n=3}^{N} \sum k(n) \prod_{l_1+\cdots+l_s=n} D_M^{2,l_i}(\imath(\cdot),(u_2-u_1)_\perp^y)^{l_i}\big(y+(u_2-u_1)_M^y\big), \quad (6)$$

where D_M^{2,l_i} are differential operators on M having the following form $D_M^{2,l_i} = (\wedge_{k=1}^{l_i}\nabla_M^{(i_k)}, \wedge_{k=1}^{l_i}\nabla_M^{(j_k)})$; the operators are applied to a product of l_i functions, i_k and j_k are numbers from 1 till l_i which have the meaning of the function number in the product on which the corresponding operator acts; the index 2 says that i_k and j_k take the same values exactly 2 times, $k(n)$ are rational functions; the second sum in the last term contains a finite number of items, the number N is chosen so that $t_1^{(N-1)\alpha} < t$.

[23] *Smolyanov O.G., Weizsäcker H.v., Wittich O.*, Brownian motion on a manifold as a limit of stepwise conditioned standard Brownian motions, Canadian Mathematical Society, Conference Proceedings, Vol. 29, 2000, pp. 589-602.

Further we pass to the main result of the third chapter.

As earlier, let M be a compact Riemannian manifold of dimension d without boundary, isometrically imbedded into $\mathbb{R}^m$. A Brownian sheet with values in $\mathbb{R}^m$ is understood as a family of m independent Brownian sheets. Let $\mathbf{W}_{t,s}$ be an m-dimensional Brownian sheet. Consider $\mathbf{W}_{t,s}$ as a process taking values in the space $C([0,1],\mathbb{R}^m)$. Denote this process by $\mathbf{W}_t$. Introduce the following notations: if E is a locally convex space (LCS) then E^t denotes $C([0,t],E)$; if $y \in C([0,1],\mathbb{R}^m)$ is a continuous function, then $\mathbb{W}^y$ denotes a distribution of the process $\mathbf{W}^y_t = y + \mathbf{W}_t$, for $\psi \in C([0,1],\mathbb{R}^m)$ we define a process $(\mathbf{W}^y_\psi)_t = \psi(t) + \mathbf{W}^y_t$. Let $\tilde{\mathbb{W}}^y_\psi$ be the distribution of this process, $\mathbb{E}_{y,\psi}$ be the expectation relative to the measure $\tilde{\mathbb{W}}^y_\psi$. Further, $U_\varepsilon(M)$ denotes the ε-neighborhood of the manifold M. We will consider $\mathbf{W}^y_\psi$ for functions y and ψ satisfying the following conditions: $y(0) \in M, \psi(0) = 0$. We prove the existence of the following limit relative to the family of bounded continuous cylinder functions, where everywhere below by a cylinder function $C([0,1]\times[0,1],\mathbb{R}^m) \to \mathbb{R}$ we understand a function f for which we can find a finite number of points $\tau_1,\dots,\tau_n$, $\xi_1,\dots,\xi_k$ and a function $\tilde{f} : \mathbb{R}^{nk} \to \mathbb{R}$ such that $f(\omega) = \tilde{f}(\omega(\tau_1,\xi_1),\omega(\tau_1,\xi_2),\dots,\omega(\tau_n,\xi_k))$. This limit defines a measure $\tilde{\mathbb{W}}^y_{M,\psi,s,t}$:

$$\int_{C([0,s],\mathbb{R}^m)^t} f(\omega)\tilde{\mathbb{W}}^y_{M,\psi,s,t}(d\omega) = \lim_{\varepsilon\to 0} \frac{\mathbb{E}_{y,\psi}\{f(\omega)\,\mathbb{I}_{\{(\mathbf{W}^y_\psi)_t(s)\in U_\varepsilon(M)\}}\}}{\tilde{\mathbb{W}}^y_\psi\{(\mathbf{W}^y_\psi)_t(s) \in U_\varepsilon(M)\}}\,. \tag{7}$$

Before we prove the existence of this limit, we consider again the discussed above process $(\mathbf{W}^z_{\psi,s})_t = \psi(t) + B^s_t$ where $\psi : [0,1] \to \mathbb{R}^m$ is a continuous function satisfying the condition $\psi(0) = 0$. The results obtained for this process will be applied for the further construction.

Some results for the process $(\mathbf{W}^z_{\psi,s})_t$. Let $\mathbb{W}^z_{\psi,s}$ denotes the distribution of the process $(\mathbf{W}^z_{\psi,s})_t$, $\mathbb{E}_{z,\psi,s}$ denotes the expectation relative this distribution.

Lemma 2. *The limit* $\int_{C([0,t],\mathbb{R}^m)} f(\omega)\mathbb{W}^z_{M,\psi,s,t}(d\omega) = \lim_{\varepsilon\to 0}\frac{\mathbb{E}_{z,\psi,s}\{f(\omega)\,\mathbb{I}_{\{(\mathbf{W}^z_{\psi,s})_t\in U_\varepsilon(M)\}}\}}{\mathbb{W}^z_{\psi,s}\{(\mathbf{W}^z_{\psi,s})_t\in U_\varepsilon(M)\}}$, *exists relative to the family of bounded continuous cylinder functions and defines a measure* $\mathbb{W}^z_{M,\psi,s,t}$.

The process corresponding to the measure $\mathbb{W}^z_{M,\psi,s,t}$ we denote by $\mathbf{W}^z_{M,\psi,s,t}$.

Outline of proof. Find the function $\tilde{f} : \mathbb{R}^{k+1} \to \mathbb{R}$ and a finite number of points

$\tau_1, \ldots, \tau_k$ such that $f(\omega) = \tilde{f}(\omega(\tau_1), \ldots, \omega(\tau_k), \omega(t))$. We have

$$\int\limits_{C([0,t],\mathbb{R}^m)} f(\omega)\mathbb{W}^z_{M,\psi,s,t}(d\omega) = \lim_{\varepsilon\to 0} \frac{\int_{C([0,t],\mathbb{R}^m)} f(\omega)\, \mathbb{I}_{\{\omega:\,\omega(t)\in U_\varepsilon(M)\}}\, \mathbb{W}^z_{\psi,s}(d\omega)}{\mathbb{W}^z_{\psi,s}\{\omega : \omega(t) \in U_\varepsilon(M)\}}$$

$$= \lim_{\varepsilon\to 0} \frac{1}{\mathbb{P}^{\mathbb{W}}(t, 0, U_\varepsilon(M - z - \psi(t)))} \cdot \int_{\mathbb{R}^m} \mathbb{P}^{\mathbb{W}}(\tau_1, 0, dx_1) \int_{\mathbb{R}^m} \mathbb{P}^{\mathbb{W}}(\tau_2 - \tau_1, x_1, dx_2) \ldots$$

$$\int_{U_\varepsilon(M-\psi(t)-z)} \mathbb{P}^{\mathbb{W}}(t - \tau_k, x_k, dx_{k+1})\tilde{f}(x_1 + z + \psi(\tau_1), \ldots, x_k + z + \psi(\tau_k), x_{k+1} + z + \psi(t)),$$

where $\mathbb{P}^{\mathbb{W}}(\tau, x, dz) = \frac{1}{(2\pi s\tau)^{\frac{m}{2}}} e^{-\frac{|z-x|^2}{2s\tau}} dz$. Since the function under the integral sign is bounded, then by the Lebesgue theorem, it is sufficient to prove the existence of the limit

$$\lim_{\varepsilon\to 0} \frac{\int\limits_{(U_\varepsilon(M-\psi(t)-z))} \tilde{f}(x_1 + z + \psi(\tau_1), \ldots, x_{k+1} + z + \psi(t))\mathbb{P}^{\mathbb{W}}(t - \tau_k, x_k, dx_{k+1})}{\mathbb{P}^{\mathbb{W}}(t, 0, U_\varepsilon(M - z - \psi(t)))}$$

$$= \lim_{\varepsilon\to 0} \frac{\int\limits_{U_\varepsilon(M-\psi(t)-z-x_k)} \tilde{f}(x_1 + z + \psi(\tau_1), \ldots, x_{k+1} + x_k + z + \psi(t))\mathbb{P}^{\mathbb{W}}(t - \tau_k, 0, dx_{k+1})}{\mathbb{P}^{\mathbb{W}}(t, 0, U_\varepsilon(M - z - \psi(t))} .$$

Denote by M_1 the manifold $M - \psi(t) - z - x_k$, by M_2 the manifold $M - \psi(t) - z$. Further, let $\lambda_\varepsilon = \frac{1}{vol_{m-d}(\varepsilon)} l|_{U_\varepsilon(M_1)}$, $\mu_\varepsilon = \frac{1}{vol_{m-d}(\varepsilon)} l|_{U_\varepsilon(M_2)}$, where l is the Lebesgue measure on $\mathbb{R}^m$. It is easy to see that the proof of the existence of this limit can be reduced to the proof of the existence of the limit $\lim_{\varepsilon\to 0} \frac{\int_{\mathbb{R}^m} g(x_{k+1})\, e^{-\frac{|x_{k+1}-x_k|^2}{2s(t-\tau_k)}} \lambda_\varepsilon(dx_{k+1})}{\int_{\mathbb{R}^m} e^{-\frac{|x_{k+1}|^2}{2st}} \mu_\varepsilon(dx_{k+1})}$, where $g : \mathbb{R}^m \to \mathbb{R}$ is another notation for the function $\tilde{f}$ where the dependence just on the last variable is expressed. It is easy to prove that as $\varepsilon \to 0$, the measures λ_ε and μ_ε converge weakly to the surface measures on M_1 and M_2, respectively. □

LEMMA 3. *The limit* (7) *exists relative to the family of bounded continuous cylinder functions.*

Outline of proof. Let $\mathbb{P}^{\tilde{\mathbb{W}}}(t, y, \Gamma) = \tilde{\mathbb{W}}^y(\omega : \omega(t) \in \Gamma)$ be the transition probability of the measure $\tilde{\mathbb{W}}^y$ where $y \in C([0, 1], \mathbb{R}^m)$. Further let the function $\tilde{f} : C([0, s], \mathbb{R}^m)^{k+1} \to \mathbb{R}$ and the finite number of points $\tau_1, \tau_2, \ldots, \tau_k$, be such that $f(\omega) = \tilde{f}(\omega(\tau_1), \omega(\tau_2), \ldots, \omega(\tau_k), \omega(t))$. The symbol π_s denotes the coordinate mapping. The proof is obtained by using a formula from the book [24] on the page 204

$$\int_{C([0,s],\mathbb{R}^m)^t} f(\omega)\tilde{\mathbb{W}}^0(d\omega) = \int\limits_{C([0,s],\mathbb{R}^m)} \mathbb{P}^{\tilde{\mathbb{W}}}(\tau_1, 0, dw_1) \int\limits_{C([0,s],\mathbb{R}^m)} \mathbb{P}^{\tilde{\mathbb{W}}}(\tau_2 - \tau_1, w_1, dw_2) \ldots \int\limits_{\pi_s^{-1}(U_\varepsilon(M-\psi(t)-y(s)))} \tilde{f}(w_1, \ldots, w_{k+1})\mathbb{P}^{\tilde{\mathbb{W}}}(t - \tau_k, w_k, dw_{k+1})$$

[24] *Ikeda, Nobuyuki; Watanabe, Shinzo*, Stochastic differential equations and diffusion processes. 2nd ed. (English) North-Holland Mathematical Library, 24. Amsterdam etc.: North-Holland; Tokyo: Kodansha Ltd. xvi, 555 p.

and applying Lemma 2 to the measure in the last integral. □

The construction of the stochastic field $\mathbf{W}_M^x$. Let f be a bounded continuous cylinder function on $C([0,s],\mathbb{R}^m)^1$, $\varphi:\mathbb{R}\to M$ be a function which is a trajectory of a Brownian motion on M such that $\varphi(0)=x$. As earlier, $\mathcal{P}_1=\{0=t_0\leqslant t_1\leqslant\ldots\leqslant t_n=1\}$ is a partition of the interval $[0,1]$ and $E^1\ni\omega=(\omega_1,\omega_2,\ldots,\omega_n)\in E^{t_1}\times E^{t_2-t_1}\times\ldots\times E^{t_n-t_{n-1}}$, where $\omega_j(t)=\omega(t_{j-1}+t)$, $\varphi_{t_{i-1}t_i}(t)=\varphi(t_{i-1}+t)-\varphi(t_{i-1})$, $t\in[0,t_j-t_{j-1}]$. Define a measure $\tilde{\mathbb{W}}^x_{M,\varphi,s,\mathcal{P}_1}$ by the formula:

$$\int\limits_{C([0,s],\mathbb{R}^m)^1} f(\omega)\tilde{\mathbb{W}}^x_{M,\varphi,s,\mathcal{P}_1}(d\omega)=\int\limits_{C([0,s],\mathbb{R}^m)^{t_1}}\tilde{\mathbb{W}}^x_{M,\varphi_{0t_1},s,t_1}(d\omega_1)\int\limits_{C([0,s],\mathbb{R}^m)^{t_2-t_1}}\tilde{\mathbb{W}}^{\omega_1(t_1)}_{M,\varphi_{t_1t_2},s,t_2-t_1}(d\omega_2)$$
$$\ldots\int\limits_{C([0,s],\mathbb{R}^m)^{t_n-t_{n-1}}}\tilde{\mathbb{W}}^{\omega_{n-1}(t_{n-1}-t_{n-2})}_{M,\varphi_{t_{n-1}t_n},s,t_n-t_{n-1}}(d\omega_n)f(\omega_1,\omega_2,\ldots,\omega_n).$$

We can check immediately that $\omega_i(t_i-t_{i-1})(0)\in M$, and hence, the measure $\tilde{\mathbb{W}}^x_{M,\varphi,s,\mathcal{P}_1}$ is well defined. Further, let $\mathcal{P}_2=\{0=s_0\leqslant s_1\leqslant\ldots\leqslant s_k=1\}$ be a partition of the interval $[0,1]$. Now we consider s as the time parameter. Instead of the symbol $\tilde{\mathbb{W}}^x_{M,\varphi,s,\mathcal{P}_1}$ we will use the symbol $\tilde{\mathbb{W}}^\varphi_{M,s,\mathcal{P}_1}$. Define the measure $\mathbb{W}^x_{M,\mathcal{P}_1,\mathcal{P}_2}$ by the formula:

$$\int\limits_{C([0,1],\mathbb{R}^m)^1} f(\omega)\mathbb{W}^x_{M,\mathcal{P}_1,\mathcal{P}_2}(d\omega)=\int\limits_{C([0,1],\mathbb{R}^m)^{s_1}}\tilde{\mathbb{W}}^x_{M,s_1,\mathcal{P}_1}(d\omega_1)\int\limits_{C([0,1],\mathbb{R}^m)^{s_2-s_1}}\tilde{\mathbb{W}}^{\omega_1(s_1)}_{M,s_2-s_1,\mathcal{P}_1}(d\omega_2)\ldots$$
$$\int\limits_{C([0,1],\mathbb{R}^m)^{s_n-s_{n-1}}}\tilde{\mathbb{W}}^{\omega_{n-1}(s_{n-1}-s_{n-2})}_{M,s_n-s_{n-1},\mathcal{P}_1}(d\omega_n)f(\omega_1,\ldots,\omega_n)\ .$$

THEOREM 7 (About a Brownian sheet). *For every $x\in M$, as $|\mathcal{P}_1|\to 0$ and $|\mathcal{P}_2|\to 0$, the distributions of $\mathbf{W}^x_{M,\mathcal{P}_1,\mathcal{P}_2}$ converge weakly relative to the family of bounded continuous cylinder functions to a measure $\mathbb{W}^x_M$. The measure $\mathbb{W}^x_M$ has the following properties:*

(i) *considered as the distribution of a* $\mathrm{C}([0,1],M)$*-valued process, $\mathbb{W}^x_M$ possesses a transition probability at time t which is the distribution of a Brownian motion on M with variance t starting at the point x;*

(ii) *considered as the distribution of an M-valued two-parameter stochastic process, $\mathbb{W}^x_M$ possesses a transition probability at time (s,t) which is the heat kernel measure on M at time st, i.e., if $f\in\mathrm{C}(M)$, then*

$$\mathbb{W}^x_M\circ\pi^{-1}_{s,t}[f]=\big(e^{-\frac{st}{2}\Delta_M}f\big)(x),$$

where $\pi_{s,t}$ is the mapping $\mathrm{C}([0,1]\times[0,1],M)\to M$, $\omega\mapsto\omega(s,t)$, *Δ_M is the Laplace–Beltrami operator on M, $e^{-\frac{\tau}{2}\Delta_M}$ is the heat semigroup.*

Outline of proof. Let the measure $\mathbb{W}^x_{M,\varphi,s,\mathcal{P}_1}$ be defined exactly as in the case of a differentiable function φ i.e. by the formula (4). Further, let first assume that there exists a function $\tilde{f} : C([0,1],\mathbb{R}^m) \to \mathbb{R}$ such that $f(\omega) = \tilde{f}(\omega(t))$. Then

$$\int\limits_{C([0,s],\mathbb{R}^m)^1} f(\omega)\,\tilde{\mathbb{W}}^{\varphi}_{M,s,\mathcal{P}_1}(d\omega) = \int\limits_{C([0,1],\mathbb{R}^m)} \tilde{f}(w)\,\tilde{\mathbb{W}}^{\varphi}_{M,s,\mathcal{P}_1} \circ \pi_s^{-1}(dw) = \int\limits_{C([0,1],\mathbb{R}^m)} \tilde{f}(w)\,\mathbb{W}^{\varphi}_{M,s,\mathcal{P}_1}(dw),$$

From this it follows:

$$\int\limits_{C([0,1],\mathbb{R}^m)^1} f(\omega)\mathbb{W}^x_{M,\mathcal{P}_1,\mathcal{P}_2}(d\omega) = \int\limits_{C([0,1],\mathbb{R}^m)} \mathbb{W}^x_{M,s_1,\mathcal{P}_1}(dw_1) \int\limits_{C([0,1],\mathbb{R}^m)} \mathbb{W}^{w_1}_{M,s_2-s_1,\mathcal{P}_1}(dw_2)\dots$$
$$\int\limits_{C([0,1],\mathbb{R}^m)} \mathbb{W}^{w_{n-2}}_{M,s_{n-1}-s_{n-2},\mathcal{P}_1}(dw_{n-1}) \int\limits_{C([0,1],\mathbb{R}^m)} \mathbb{W}^{w_{n-1}}_{M,s_n-s_{n-1},\mathcal{P}_1}(dw_n)\tilde{f}(w_n).$$

Consider an integral of the form $\int_{C([0,t],\mathbb{R}^m)} g(\omega)\mathbb{W}^z_{M,\psi,s,t}(d\omega)$, where the function $g \in C([0,t],\mathbb{R}^m)$ can be presented in the form $g(\omega) = \tilde{g}(\omega(t))$, $\tilde{g} \in C(\mathbb{R})$. After simple calculations we get

$$\int\limits_{C([0,t],\mathbb{R}^m)} g(\omega)\mathbb{W}^z_{M,\psi,s,t}(d\omega) = \lim_{\varepsilon\to 0} \frac{\int\limits_{C([0,t],\mathbb{R}^m)} g(\omega)\,\mathbb{I}_{\{\omega:\,\omega(t)\in U_\varepsilon(M)\}}(\omega)\,\mathbb{W}^z_{\psi,s}(d\omega)}{\mathbb{W}^z_{\psi,s}\{\omega : \omega(t) \in U_\varepsilon(M)\}}$$
$$= \frac{\int\limits_M e^{-\frac{|x_1-z-\psi(t)|^2}{2ts}}\tilde{g}(x_1)\,\lambda_M(dx_1)}{\int\limits_M e^{-\frac{|x_1-z-\psi(t)|^2}{2ts}}\lambda_M(dx_1)}.$$

Assume first that the function f is such that there exist a function $p : \mathbb{R}^m \to \mathbb{R}$ and numbers $t, s \in [0,1]$ such that $f(\omega) = p(\omega(t,s))$. The integral

$\int_{C([0,1],\mathbb{R}^m)} f(\omega)\mathbb{W}^x_{M,\mathcal{P}_1,\mathcal{P}_2}(d\omega)$ has the form:

$$\frac{\int_M e^{-\frac{|x_1-x|^2}{2\Delta s_1\Delta t_1}}dx_1}{\int_M e^{-\frac{|\bar{x}_1-x|^2}{2\Delta s_1\Delta t_1}}d\bar{x}_1}\cdots\frac{\int_M e^{-\frac{|x_{n-1}-x_{n-2}|^2}{2\Delta s_1\Delta t_{n-1}}}dx_{n-1}}{\int_M e^{-\frac{|\bar{x}_{n-1}-x_{n-2}|^2}{2\Delta s_1\Delta t_{n-1}}}d\bar{x}_{n-1}}\frac{\int_M e^{-\frac{|x_n-x_{n-1}|^2}{2\Delta s_1\Delta t_n}}dx_n}{\int_M e^{-\frac{|\bar{x}_n-x_{n-1}|^2}{2\Delta s_1\Delta t_n}}d\bar{x}_n}$$

$$\frac{\int_M e^{-\frac{|y_1-x_1|^2}{2\Delta s_2\Delta t_1}}dy_1}{\int_M e^{-\frac{|\bar{y}_1-x_1|^2}{2\Delta s_2\Delta t_1}}d\bar{y}_1}\cdots\frac{\int_M e^{-\frac{|y_{n-1}-y_{n-2}-x_{n-1}+x_{n-2}|^2}{2\Delta s_2\Delta t_{n-1}}}dy_{n-1}}{\int_M e^{-\frac{|\bar{y}_{n-1}-y_{n-2}-x_{n-1}+x_{n-2}|^2}{2\Delta s_2\Delta t_{n-1}}}d\bar{y}_{n-1}}\frac{\int_M e^{-\frac{|y_n-y_{n-1}-x_n+x_{n-1}|^2}{2\Delta s_2\Delta t_n}}dy_n}{\int_M e^{-\frac{|\bar{y}_n-y_{n-1}-x_n+x_{n-1}|^2}{2\Delta s_2\Delta t_n}}d\bar{y}_n}$$

$$\cdots$$

$$\frac{\int_M e^{-\frac{|u_1-z_1|^2}{2\Delta s_{k-1}\Delta t_1}}du_1}{\int_M e^{-\frac{|\bar{u}_1-z_1|^2}{2\Delta s_{k-1}\Delta t_1}}d\bar{u}_1}\cdots\frac{\int_M e^{-\frac{|u_{n-1}-u_{n-2}-z_{n-1}+z_{n-2}|^2}{2\Delta s_{k-1}\Delta t_{n-1}}}du_{n-1}}{\int_M e^{-\frac{|\bar{u}_{n-1}-u_{n-2}-z_{n-1}+z_{n-2}|^2}{2\Delta s_{k-1}\Delta t_{n-1}}}d\bar{u}_{n-1}}\frac{\int_M e^{-\frac{|u_n-u_{n-1}-z_n+z_{n-1}|^2}{2\Delta s_{k-1}\Delta t_n}}du_n}{\int_M e^{-\frac{|\bar{u}_n-u_{n-1}-z_n+z_{n-1}|^2}{2\Delta s_{k-1}\Delta t_n}}d\bar{u}_n}$$

$$\frac{\int_M e^{-\frac{|v_1-u_1|^2}{2\Delta s_k\Delta t_1}}dv_1}{\int_M e^{-\frac{|\bar{v}_1-u_1|^2}{2\Delta s_k\Delta t_1}}d\bar{v}_1}\cdots\frac{\int_M e^{-\frac{|v_{n-1}-v_{n-2}-u_{n-1}+u_{n-2}|^2}{2\Delta s_k\Delta t_{n-1}}}dv_{n-1}}{\int_M e^{-\frac{|\bar{v}_{n-1}-v_{n-2}-u_{n-1}+u_{n-2}|^2}{2\Delta s_k\Delta t_{n-1}}}d\bar{v}_{n-1}}\frac{\int_M e^{-\frac{|v_n-v_{n-1}-u_n+u_{n-1}|^2}{2\Delta s_k\Delta t_n}}p(v_n)dv_n}{\int_M e^{-\frac{|\bar{v}_n-v_{n-1}-u_n+u_{n-1}|^2}{2\Delta s_k\Delta t_n}}d\bar{v}_n},$$

where $\Delta t_i = t_i - t_{i-1}$, $\Delta s_j = s_j - s_{j-1}$, for simplicity of notations instead of $\lambda_M(dz)$ we use dz. We also assumed here that $t_n = t$, $s_k = s$. Denote this integral by $I(\mathcal{P}_1,\mathcal{P}_2,p)$.

Lemma 4. *$I(\mathcal{P}_1,\mathcal{P}_2,p)$ converges to $e^{-\frac{st}{2}\Delta_M}p$, as the meshes $|\mathcal{P}_1|$ and $|\mathcal{P}_2|$ tend to zero.*

Outline of proof. We choose $0<\alpha<\tilde{\alpha}<\frac{1}{2}$, $\Delta\alpha = \tilde{\alpha}-\alpha$. In the integral of the form $\frac{\int_M e^{-\frac{|x_i-x_{i-1}|^2}{2\Delta s_1\Delta t_i}}dx_i}{\int_M e^{-\frac{|\bar{x}_i-x_{i-1}|^2}{2\Delta s_1\Delta t_i}}d\bar{x}_i}$, at the i-th place in the first line we replace the integration over the whole manifold with the integration over a neighborhood of the point x_{i-1} of radius $|\mathcal{P}_2|^{\tilde{\alpha}}(\Delta t_i)^{\tilde{\alpha}}$. Here we get a rest term not exceeding $\frac{C}{(2\pi\Delta s_1\Delta t_i)^{\frac{d}{2}}}e^{-\frac{1}{(\Delta s_1\Delta t_i)^{1-2\tilde{\alpha}}}}$, C is a constant. For the i-th integral of the second line we have $|x_i-x_{i-1}|<|\mathcal{P}_2|^{\tilde{\alpha}}(\Delta t_i)^{\tilde{\alpha}}$. Further, we choose a a neighborhood of the point y_{i-1} on M for all points y_i of which the inequality $|y_i-y_{i-1}|<2^{\Delta\alpha}|\mathcal{P}_2|^{\tilde{\alpha}}(\Delta t_i)^{\tilde{\alpha}}$ holds. Again we have a rest term not exceeding $\frac{C}{(2\pi\Delta s_2\Delta t_i)^{\frac{d}{2}}}e^{-\frac{C_1}{(\Delta s_2\Delta t_i)^{1-2\tilde{\alpha}}}}$, C, C_1 are constants. We continue fixing neighborhoods by the above way. Finally we will consider the ith integral of the last line for points u_i and u_{i-1} satisfying $|u_i-u_{i-1}|<(K-1)^{\Delta\alpha}|\mathcal{P}_2|^{\tilde{\alpha}}(\Delta t_i)^{\tilde{\alpha}}<|\mathcal{P}_2|^{\alpha}(\Delta t_i)^{\alpha}$, where the inequality holds for sufficiently small mesh of $|\mathcal{P}_1|$ where $(K|\mathcal{P}_2|\Delta t_i)^{\Delta\alpha}<1$. Further, starting from the last integral of the last line, we apply the asymptotic of Proposition 2. The asymptotic is being applied to each integral subsequently. Terms containing expressions of the form u_i-u_{i-1} disappear either after applying of the first line of integrals or earlier. Thus we get an asymptotic decomposition for the integral $I(\mathcal{P}_1, \mathcal{P}_2, p)$. We compare this decomposition with the decomposition for $e^{-\frac{st}{2}\Delta_M} = e^{-\frac{\Delta s_1 \Delta t_1}{2}\Delta_M} \cdots e^{-\frac{\Delta s_k \Delta t_n}{2}\Delta_M}$, where we apply the asymptotic $e^{-\frac{\Delta s_i \Delta t_j}{2}\Delta_M} = 1 - \frac{\Delta s_i \Delta t_j}{2}\Delta_M + O((\Delta s_i \Delta t_j)^{\frac{3}{2}})$ to each of these exponents. Up to terms converging to zero as $|\mathcal{P}_1| \to 0$, $|\mathcal{P}_2| \to 0$, the decompositions will coincide. □

For a function f depending on ω at several points, say, at the points $\xi_i \in [0, s]$ and $\tau_j \in [0, t]$, the form of the integral $I(\mathcal{P}_1, \mathcal{P}_2, p)$ will be the same, and the convergence can be proved analogously. The function $p = (x_{11}, \dots x_{ij}, \dots x_{lr})$ is assumed to satisfy the equality $f(\omega) = p(\omega_{11}(\xi_1, \tau_1), \dots, \omega_{kl}(\xi_l, \tau_r))$, here ω_{ij} is defined on $[0, \xi_i - \xi_{i-1}] \times [0, \tau_j - \tau_{j-1}]$ by $\omega_{ij}(s,t) = \omega(\xi_{i-1} + s, \tau_{j-1} + t)$. The integral converges to a product of operators of the form $e^{-\frac{\Delta \xi_i \Delta \tau_j}{2}\Delta_M}$, each of the operators is applied to the function which is obtained from the function p by fixing all variable besides the variable at the position ij. □

COROLLARY 2. *Let M be a compact Lie group. Then, $\mathbf{W}_M^x$ being considered as a $C([0,1], M)$-valued process, coincides with the Brownian motion constructed by Malliavin in* [25].

Outline of proof. The proof follows from Theorem 7 and Theorem 2.15 from the paper [26] (Theorem 2.15 from the paper [25] is proved also in Lemma 3.3 in the paper [27]). □

Acknowledgment

The author expresses the gratitude to her Ph.D. thesis advisor Professor O. G. Smolyanov for his attention to the work, very useful discussions and remarks, his significant impact into formation of author's scientific interests, and for emotional support.

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2. *Shamarova, É. Yu*, Chernoff's theorem for evolution families of operators, Proceedings of the conference "Lomonosov's readings", 2003, pp. 457 – 460 (in Russian)
3. *Shamarova, Eh. Yu*, Construction of a Brownian sheet with values in a compact Riemannian manifold. (Russian, English) Math. Notes 76, No.4, 590-596 (2004); translation from Mat. Zametki 76, No.4, 635-640 (2004).